Gravitic Motion and the Inverse-Square Law

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Abstract. We show that random bombardment by G particles, moving in all directions without interacting, pushes any two bodies together with a force that falls off exactly as the inverse square of their separation. The result is established for gravitation, the G1 sea acting on ordinary matter, with G1 particles at the speed of light; the derivation refers only to a sea of particles and to bodies that partly block it. Two bodies each shadow the other and are pushed together. For opaque spheres of radii R_1 and R_2 a distance d apart in a sea of intensity $\rho_G = nm_Gv^2$, the force is $F = (\pi\rho_G/4)\,R_1^2R_2^2/d^2$, equal and opposite on the two bodies. The inverse square is exact for every $d > R_1$, and a Monte-Carlo simulation of random impacts reproduces the law and its symmetry. As a marked remark, if the four universal motions are right that gravitic motion occurs at every level, the same law read one level down takes the form of Coulomb's law, with charge identified as a blocking area in a finer sea, so that charge quantisation follows from counting identical blockers; repulsion is not supplied by shadowing and is left to magnetic motion. Partial transparency changes what the force couples to, area when opaque and mass when transparent, without changing the inverse square. The opaque reading gives three values of G spanning a factor of 446 across three solar-system pairs and is excluded; in the transparent regime $G = \kappa^2\rho_{G1}/4\pi$ is a composite constant. We quantify the open problems: the shielding bound requires a G1 sea of at least 4×10^{20} kg m^{-3}; orbital drag cancels exactly for each thin layer under a $1/w$ cross-section, provided deflected particles leave isotropically in the sea's frame, but not for a self-shielding body; and heating is answered in collision terms.

Keywords: push gravity; Le Sage gravitation; inverse-square law; gravitational shielding; Coulomb's law; four universal motions

1 The mechanism

Two large bodies sit in a sea of G particles flying in every direction at high speed: the speed of light in the G1 sea, and faster still in the seas of the levels below. The particles are so sparse in their mutual interaction that they never scatter off one another; they fly in straight lines until they meet ordinary matter.

Each body alone is bombarded equally from all sides, and goes nowhere. But each body blocks some of what passes through it. So body 2 receives slightly *less* momentum from the direction of body 1 than from the opposite direction — body 1 has removed a cone of sky from body 2's view. The unopposed bombardment from the far side pushes body 2 toward body 1. By the identical argument body 1 is pushed toward body 2.

The two bodies are not pulled together. **They are pushed together by the flux that is not blocked.** Everything below is the quantitative statement of that sentence.

A note on level. Nothing in that description mentions a scale. It does not say how large the bodies are, what they are made of, or how much they weigh — only that a sea of randomly moving particles is partly blocked by two objects. The four motions [1, 2] hold that gravitic motion occurs at *every* level of the universe, of which there are infinitely many, and the derivation below is written to match: it speaks throughout of **G particles** and a sea of intensity ρ_G, never of one privileged level. Every result in §3–§7 would therefore apply unchanged at any level where its conditions hold. This paper does not test the claim that such levels exist; its results are established for level 1.

Where a level must be named, the seas are written $G1, G2, G3, \ldots$. Level 1 — the G1 sea acting on ordinary matter — is what we call gravitation, and §7.3 and §9 work out its numbers. Level 2 —

the G2 sea acting on the constituents of matter — is what the framework identifies with electricity, and §8 sets out, as a marked remark, what follows if that identification holds. The inverse-square law itself is a property of gravitic motion as such, not of any one level.

2 Postulates

P1 — The G sea. Space is permeated by G particles of number density n and mass m_G, travelling at speed v in random directions. The direction distribution is isotropic and the sea is homogeneous over the scales considered. No level is specified: n, m_G, v and the bodies below belong to whichever level is under discussion.

P2 — Negligible self-interaction. G particles do not interact with one another. There is no pressure, no thermalisation, no mean free path within the sea; each particle flies straight until it meets matter.

This is not a technical convenience. It is what makes the mechanism work at all. In an ordinary self-interacting gas a shadow is erased within a few mean free paths, because particles scatter sideways into the depleted region. A non-interacting flux cannot do that: shadows in it are sharp and persist undegraded over any distance, which is exactly what a long-range force requires.

There is direct evidence for this contrast, from a system where the same shadowing mechanism operates in a *dense* bath. Two large spheres immersed in a fluid of small hard spheres attract each other because each shadows the other from bombardment — the depletion or Asakura–Oosawa force, which has been measured in particle-level simulation [3]. That force is real, and it is emphatically *not* inverse-square: it is a contact-range interaction that dies within about one small-sphere diameter, precisely because the bath is collisional and refills the shadow. The same mechanism gives a short-range force in a dense medium and a $1/d^2$ force in a collisionless one. P2 is the whole difference.

P3 — Interaction with matter. A G particle crossing ordinary matter may be deflected out of its line of travel. It is not stopped: it leaves again in a new direction (§9.4). For the shadow, what matters is only that it no longer continues along its original line to the bodies beyond. §3–5 treat the limiting case of a body that deflects every particle crossing it; §7 treats partial transparency and shows the two cases are limits of one formula.

The interaction strength may depend on the *relative* speed w between particle and body. For bodies at rest in the sea — which is the case throughout §3–§8 — every incoming particle has $w = v$, so the cross-section is a single number and nothing in those sections is affected. The velocity dependence matters only for a body in motion, and §9.2 shows that it matters there decisively.

P4 — The push. A deflected particle pushes the body that deflected it. Averaged over the random directions in which deflected particles leave, the push is $m_G v$ along the particle's original direction of travel.

P5 — Levels and speeds. In the four motions, the universe has infinitely many levels of scale, and gravitic motion occurs at all of them. The results of this paper use only level 1; the other levels enter only through the speed ordering below and the remark of §8. Each level has its own sea of randomly moving G particles — written $G1, G2, G3, \ldots$ when a level must be named — and its own bodies, built from the particles of the level below, which block some of what passes through them. P1–P4 are stated for a generic level and are assumed to hold at every one. The one thing that changes from level to level, apart from the particles' size, is their speed. G1 particles travel at

exactly the speed of light, $v_1 = c$, and each sea is faster than the one above it,

$$v_1 = c \;<\; v_2 \;<\; v_3 \;<\; \cdots,$$

because the particles of each level must be fast enough to curve the paths of the particles of the level above. Sections 3–7 are therefore derived once and apply everywhere; §7.3 and §9 then specialise to level 1 (gravitation), and the remark of §8 reads them at level 2 (electricity).

3 The G momentum flux

Let $\hat{n}$ be a unit direction. The number of G particles per unit volume travelling within solid angle $d\Omega$ about $\hat{n}$ is $n\,d\Omega/4\pi$, so the particle current density measured perpendicular to the beam is

$$dJ(\hat{n}) = \frac{nv}{4\pi}\,d\Omega \qquad [\mathrm{m^{-2}s^{-1}}],$$

and the momentum it delivers per unit perpendicular area per unit time is

$$dI(\hat{n}) = m_G v\,dJ = \frac{n\,m_G v^2}{4\pi}\,d\Omega.$$

Name the combination that governs everything:

$$\boxed{\rho_G \;\equiv\; n\,m_G\,v^2} \qquad [\mathrm{J\,m^{-3}} = \mathrm{Pa}],$$

the **G flux intensity** — dimensionally an energy density, or equivalently a pressure. It is the only way the sea enters the force law, so the whole model has one free parameter at this stage. Thus

$$dI(\hat{n}) = \frac{\rho_G}{4\pi}\,d\Omega. \tag{1}$$

4 One body alone feels nothing

A sphere of radius R presents the same projected area πR^2 to every direction. The momentum it receives per unit time from directions within $d\Omega$ about $\hat{n}$ is $(\rho_G/4\pi)\,\pi R^2\,d\Omega$, directed along $\hat{n}$. Integrating over the whole sky,

$$\mathbf{F} = \frac{\rho_G \pi R^2}{4\pi}\int_{4\pi} \hat{n}\,d\Omega = \mathbf{0}, \tag{2}$$

because $\int \hat{n}\,d\Omega = 0$ over the full sphere. A lone body is hammered from all sides and stays put.

This is the key structural fact. The force cannot come from the bombardment itself; it can only come from bombardment that is *missing*. Gravitation, in this model, is a subtraction.

5 Two bodies: the inverse-square law

5.1 Setup

Two spheres, radii R_1 and R_2, centres separated by d, both opaque to the sea (P3): every G particle that meets either body is deflected. Let $\hat{r}$ point from body 1 to body 2.

5.2 The blocked cone

Seen from body 2, body 1 covers a disc on the sky. Its angular radius $\theta_{\max}$ satisfies

$$\sin\theta_{\max} = \frac{R_1}{d}. \tag{3}$$

Every G particle that would have arrived at body 2 from inside that cone has instead been deflected by body 1. Those directions are missing from body 2's sky.

5.3 The missing momentum

By (1) and §4, the momentum body 2 would have received from the cone, per unit time, is

$$\Delta\mathbf{p} \;=\; \frac{\rho_G}{4\pi}\,\pi R_2^2 \int_{\text{cone}} \hat{n}\, d\Omega.$$

Only the axial component survives by symmetry, so with $\hat{n}\cdot\hat{r} = \cos\theta$ and $d\Omega = 2\pi\sin\theta\, d\theta$,

$$\int_{\text{cone}} \cos\theta\, d\Omega \;=\; 2\pi\int_0^{\theta_{\max}} \sin\theta\cos\theta\, d\theta \;=\; \pi\sin^2\theta_{\max} \;=\; \frac{\pi R_1^2}{d^2}. \tag{4}$$

This is where the inverse square comes from, and it is exact. No expansion, no far-field approximation, no $d \gg R_1$ assumption: (4) holds identically for every $d > R_1$. Direct numerical integration confirms it to twelve significant figures from $d/R_1 = 1.0000001$ (bodies in contact) to $d/R_1 = 5000$:

Table 1: Numerical check of the cone integral (4) against $\pi R_1^2/d^2$, from contact to $d/R_1 = 5000$.

d/R_1	$\int_{\text{cone}} \cos\theta\, d\Omega$	$\pi R_1^2/d^2$	ratio
1.0000001	3.141592025271	3.141592025271	1.000000000000
1.1	2.596357564950	2.596357564950	1.000000000000
2	$7.853981633974 \times 10^{-1}$	$7.853981633974 \times 10^{-1}$	1.000000000000
50	$1.256637061436 \times 10^{-3}$	$1.256637061436 \times 10^{-3}$	1.000000000000
5000	$1.256637061436 \times 10^{-7}$	$1.256637061436 \times 10^{-7}$	1.000000000000

The obliquity factor $\cos\theta$ and the solid-angle element conspire to cancel exactly, leaving pure $1/d^2$.

5.4 The force

The net force on body 2 is the negative of the missing momentum flux — it is pushed *toward* the shadow:

> **Theorem 1 (the gravitic two-body force).** *Two opaque spheres of radii R_1 and R_2, immersed in an isotropic collisionless G flux of intensity ρ_G and separated by d, are pushed together along their line of centres with force*

$$\boxed{F \;=\; \frac{\rho_G}{4\pi}\cdot \underbrace{\frac{\pi R_1^2}{d^2}}_{\text{blocked solid angle}} \cdot \underbrace{\pi R_2^2}_{\text{target area}} \;=\; \frac{\pi\rho_G}{4}\,\frac{R_1^2 R_2^2}{d^2}} \tag{5}$$

5.5 The force on *both* bodies

Body 1 is treated by the identical argument with the labels exchanged: body 2 subtends solid angle $\pi R_2^2/d^2$ at body 1, and body 1 presents target area πR_1^2. The product is the same:

$$F_{1\leftarrow 2} = \frac{\rho_G}{4\pi}\cdot\frac{\pi R_2^2}{d^2}\cdot \pi R_1^2 = \frac{\pi\rho_G}{4}\frac{R_1^2R_2^2}{d^2} = F_{2\leftarrow 1}. \tag{6}$$

So the two forces are **equal in magnitude and opposite in direction** — Newton's third law, obtained rather than assumed, from the symmetry of the expression under $1 \leftrightarrow 2$. The net force on the pair as a whole is zero; the pair simply closes on itself. Nothing external pushes the system anywhere, which is what a mutual force must do.

5.6 Why the inverse square is unavoidable

Strip the derivation to its skeleton and the reason becomes plain:

1. A body of blocking area A_1 at distance d removes a patch of sky of angular area A_1/d^2. That is the definition of solid angle in three dimensions.
2. The momentum arriving per unit solid angle is a constant, $\rho_G/4\pi$, set by the sea alone.
3. The second body collects that deficit over its own blocking area A_2.

Force = (intensity per steradian) × (steradians removed) × (collecting area) $= \dfrac{\rho_G}{4\pi}\dfrac{A_1A_2}{d^2}$.

The only d-dependence anywhere is the solid angle, and solid angle in three dimensions goes as $1/d^2$ because the surface of a sphere goes as d^2. **Random bombardment in three-dimensional space cannot produce anything other than an inverse-square law.** In D dimensions the same argument gives $1/d^{\,D-1}$.

6 Simulation of the bombardment

To check the result against the physical process rather than the algebra, we simulated it directly: millions of G particles with isotropically random directions and random impact points, exact

ray–sphere intersection to determine whether body 1 lies upstream on each ray, and summation of the deposited momentum. (Variance reduction uses the analytic vanishing of the unblocked isotropic part, §4.) Code in Appendix B.

Table 2: Monte-Carlo simulation of the bombardment compared with (5); simulated values with their statistical uncertainty.

R_1	R_2	d	F simulated	$\frac{\pi}{4}R_1^2R_2^2/d^2$	ratio
1	2	20	-7.998×10^{-3} $(\pm1.6\times10^{-4})$	-7.854×10^{-3}	1.018
1	2	60	-8.511×10^{-4} $(\pm5.2\times10^{-5})$	-8.727×10^{-4}	0.975
2	1	60	-8.833×10^{-4} $(\pm2.6\times10^{-5})$	-8.727×10^{-4}	1.012
3	0.5	40	-1.1054×10^{-3} $(\pm1.5\times10^{-5})$	-1.1045×10^{-3}	1.001

The simulated force is attractive, agrees with (5) within Monte-Carlo error, and follows $1/d^2$ across the range. Swapping the labels ($R_1 = 1, R_2 = 2$ versus $R_1 = 2, R_2 = 1$) gives the same force to within error, confirming (6) directly from the simulated impacts.

7 Partial transparency

7.1 The blocking area of a partly transparent body

§5 took the bodies to stop every particle crossing them. Real bodies may stop only a fraction, and the model should say what happens in between. Let the probability that a G particle is deflected while crossing column density $\sigma = \int \rho\, d\ell$ be $1 - e^{-\kappa\sigma}$, with κ the opacity of the bodies to their own level's sea, in $\mathrm{m^2kg^{-1}}$. (At level 1, κ is the opacity of ordinary matter to G1 and turns out to be the Majorana shielding coefficient, §9.3; at level 2 it is written κ_2, §8.3.)

Then the blocking area of a uniform sphere is the geometric disc weighted by stopping probability,

$$S = 2\pi\int_0^R b\left(1 - e^{-2\kappa\rho\sqrt{R^2-b^2}}\right) db \;=\; \pi R^2\,\Phi(\tau), \qquad \Phi(\tau) = 1 - \frac{2}{\tau^2}\Big[1 - (1+\tau)e^{-\tau}\Big], \tag{7}$$

with $\tau \equiv 2\kappa\rho R$ the optical depth through the centre. **This function is not new.** Writing $\tau = 2u$, Φ is algebraically identical to the apparent-mass function $\Psi(u)$ introduced by Majorana [4] and re-derived by Poincaré [5] and by Radzievskii and Kagalnikova [6]: $\Phi(2u) = \frac{4}{3}u\,\Psi(u)$, verified here to machine precision. What is written above is that function in effective-blocking-area variables. See §10. The two-body law of §5 carries over unchanged with $\pi R_i^2 \to S_i$:

$$\boxed{F = \frac{\rho_G}{4\pi}\,\frac{S_1 S_2}{d^2} = \frac{\pi\rho_G}{4}\,\frac{R_1^2R_2^2\,\Phi(\tau_1)\Phi(\tau_2)}{d^2}} \tag{8}$$

7.2 The two regimes

The inverse square survives untouched. Φ depends on the bodies and on κ; it contains no d. So $1/d^2$ holds at every opacity from perfectly transparent to perfectly opaque — confirmed by

simulation in all three regimes (Appendix B). What the opacity changes is not the distance law but what the force couples to:

$$\Phi(\tau) \to 1 \ (\tau \gg 1) \ \Rightarrow \ S = \pi R^2; \qquad \Phi(\tau) \to \tfrac{2}{3}\tau \ (\tau \ll 1) \ \Rightarrow \ S = \kappa\, m. \tag{9}$$

The approach to the transparent limit is worth recording, because it is the model's first departure from exact Newtonian behaviour. Expanding (7),

$$\Phi(\tau) = \tfrac{2}{3}\tau - \tfrac{1}{4}\tau^2 + \tfrac{1}{15}\tau^3 - \cdots ,$$

so that a body's *effective gravitational mass* $m_{\text{grav}} \equiv S/\kappa$ falls short of its inertial mass by

$$\boxed{\frac{m_{\text{grav}}}{m_{\text{inertial}}} \;=\; 1 - \tfrac{3}{4}\,\kappa\rho R + O\Big((\kappa\rho R)^2\Big)} \tag{10}$$

verified numerically to four significant figures across eight decades of κ (Appendix B). This result is also known: it is the small-u expansion of Majorana's Ψ, and appears in this form, and with the same equivalence-principle reading, in Nali [7]. Physically, the outer layers of a body shield its interior, so the interior contributes less than its share of the shadow. Because the deficit depends on ρR — a body's size and density, not merely its mass — (10) is a violation of the equivalence principle. It is not, however, an independent test of the model: the lunar-ranging bound on κ is itself obtained from an equivalence-principle test, so the size of this deficit is fixed by the same measurement (§9.3).

Table 3: The opaque and transparent limits of the blocking area and the resulting force law.

Regime	Blocking areas	Force law
Opaque ($\tau \gg 1$)	$S_i = \pi R_i^2$	$F = \dfrac{\pi\rho_G}{4}\dfrac{R_1^2 R_2^2}{d^2}$ — Theorem 1
Transparent ($\tau \ll 1$)	$S_i = \kappa m_i$	$F = \dfrac{\kappa^2\rho_G}{4\pi}\dfrac{m_1 m_2}{d^2}$ — Newton's law, $G = \dfrac{\kappa^2\rho_G}{4\pi}$

This is a satisfying result in its own right: one mechanism, one formula, and the celebrated form Gm_1m_2/d^2 appears as the transparent limit of the same shadowing that gives $R_1^2R_2^2/d^2$ in the opaque limit. In the transparent limit the law is not merely inverse-square but fully Newtonian — the product of masses, strict attraction, the third law, linear superposition, and the shell theorem all follow, for arbitrary shapes at arbitrary separation.

7.3 Newton's constant, and which regime is real

Everything to this point has been level-free, as P5 requires: §3–§7.2 speak only of G particles, a sea of intensity ρ_G, and bodies that block it, and every result holds at all levels of the universe at once. This subsection is the first that does not. Here we specialise to **level 1** — the G1 sea acting on ordinary matter, which is what we call gravitation — and write ρ_{G1} for its intensity, so that the constant in question is Newton's. Level 2 is taken up in §8, and the tower continues without end.

The two regimes differ not only in what the force couples to, but in what *kind of constant* multiplies it — and following that through settles the question.

In the opaque regime there is no G. The coefficient in $F = (\pi\rho_{G1}/4)R_1^2R_2^2/d^2$ has units of pressure, $\mathrm{J\,m^{-3}}$, not $\mathrm{m^3kg^{-1}s^{-2}}$. It is not a different value of Newton's constant; it is a different kind of constant altogether. Newton's constant appears only if one insists on rewriting the force in terms of masses — and it is then not constant. For two uniform spheres, using $m = \frac{4}{3}\pi R^3\rho$,

$$G_{\text{eff}} \;\equiv\; \frac{Fd^2}{m_1m_2} \;=\; \frac{\pi\rho_{G1}}{4}\frac{R_1^2R_2^2}{m_1m_2} \;=\; \frac{9\,\rho_{G1}}{64\pi\,(\rho_1R_1)(\rho_2R_2)}. \tag{11}$$

G_{eff} falls as the inverse product of the two bodies' **column densities**. Denser or larger bodies would gravitate with a smaller effective G.

This is immediately falsified, and it takes no laboratory to do it — the solar system refutes itself. Calibrating ρ_{G1} on the Earth–Moon system and then applying (11) to other pairs:

Table 4: Effective Newtonian constant implied by the opaque regime, calibrated on the Earth–Moon system.

pair	$(\rho_1R_1)(\rho_2R_2)$	G_{eff}	G_{eff}/G
Earth–Moon (calibration)	2.04×10^{20}	6.674×10^{-11}	1
Earth–Sun	3.44×10^{22}	3.95×10^{-13}	1/169
Sun–Jupiter	9.08×10^{22}	1.50×10^{-13}	1/446

Three pairs, three values of G, spanning a factor of 446 — using nothing but bodies whose masses and orbits have been known for centuries. The opaque regime cannot fit the solar system to itself. (For completeness, the same calibration predicts 2.4×10^{9} N between two 1-metre iron spheres 10 metres apart against a Cavendish value of 7.1×10^{-4} N, but the astronomical comparison is the decisive one.)

In the transparent regime G is a genuine constant, and a composite one. From (9),

$$\boxed{\,G \;=\; \frac{\kappa^2\,\rho_{G1}}{4\pi}\,} \tag{12}$$

with the units working exactly: $[\kappa^2][\rho_{G1}] = (\mathrm{m^4kg^{-2}})(\mathrm{kg\,m^{-1}s^{-2}}) = \mathrm{m^3kg^{-1}s^{-2}}$. This identification also has a literature: it is Nali's (2026) $G = \Lambda^2 w_e/4\pi$, and Fedosin [8] gives the equivalent in cross-section-per-nucleon form. See §10. Newton's constant decomposes into three factors — a solid angle, a property of matter (κ), and a property of the vacuum (ρ_{G1}) — and κ carries all of the dimensional content that the sea does not supply. The sea alone gives only a pressure; κ^2 is what converts a pressure into a gravitational constant.

Three consequences are worth recording here.

The universality of G becomes a derived fact rather than an axiom. G is the same for every pair of bodies *only because* matter is optically thin; equation (11) shows that any appreciable opacity makes it body-dependent. Every precision confirmation that G is universal is therefore, in this model, a measurement of κ — the same experiments discussed in §9.3, seen from the other side.

Gravity alone can never determine the sea. Equation (11) is one equation in two unknowns, and κ has only an upper bound, so ρ_{G1} has only a lower bound. No gravitational measurement whatever can fix ρ_{G1}. But a single *detection* of shielding — a nonzero κ — would fix it exactly, and with it n_1m_{G1}. Shielding experiments are the only instrument that can see the sea directly.

G **varies if and only if the sea varies**, and exactly proportionally: $\dot{G}/G = \dot{\rho}_{G1}/\rho_{G1}$, since κ is a property of matter. Lunar laser ranging bounds $|\dot{G}/G|$ at order 10^{-13} yr^{-1}, so the G1 sea has been stable to roughly 0.1% over the age of the universe.

The verdict. The opaque regime gives the cleanest derivation and the most economical sea, but it is excluded by three centuries of celestial mechanics. Ordinary matter is in the transparent regime, $\tau \lll 1$, and the operative law is Newton's with $G = \kappa^2\rho_{G1}/4\pi$. §9.3 gives the bound on κ and what it costs the model.

7.4 The field inside a body

Everything so far has placed both bodies outside each other. A theory of gravitation must also say what happens *inside* matter — Newton's interior solution, $g = GM_{\rm enc}(r)/r^2$, rising linearly from zero at the centre of a uniform sphere. This is where transparency stops being a technical parameter and becomes the physical content of the model.

Why there is an interior field at all. A G particle arriving at an interior point has been attenuated by the column of material it crossed *before* reaching that point, and by nothing else; matter beyond the point is irrelevant to what arrives there. For a point at radius r inside a uniform sphere of radius R and density ρ, the upstream path length along a ray whose direction makes angle θ with the outward radial is

$$t(\theta) = r\cos\theta + \sqrt{R^2 - r^2\sin^2\theta}, \qquad \sigma(\theta) = \rho\, t(\theta). \tag{13}$$

Rays arriving from the near side have crossed a short column and are barely attenuated; rays from the far side have crossed a long one and are weaker. The interior point is therefore pushed **toward the centre** — by the same subtraction as before, now operating within the body.

The interior field is exactly Newtonian. Taking the radial component and linearising in the optical depth, with $u = \cos\theta$,

$$\int_{4\pi} \cos\theta\,(1 - e^{-\kappa\sigma})\, d\Omega \;\simeq\; 2\pi\kappa\rho \int_{-1}^{1} u\Big[\, ru + \sqrt{R^2 - r^2 + r^2u^2}\,\Big] du \;=\; \frac{4\pi\kappa\rho\, r}{3}. \tag{14}$$

The second term vanishes identically: u times a function of u^2 is odd over a symmetric interval. Only the first survives, giving $\frac{2}{3}r$, and the force on a test mass m becomes

$$F = \frac{\kappa m \rho_G}{4\pi} \cdot \frac{4\pi\kappa\rho r}{3} = \frac{\kappa^2\rho_G}{4\pi}\,\frac{m\, M_{\rm enc}(r)}{r^2}, \qquad M_{\rm enc}(r) = \tfrac{4}{3}\pi r^3\rho,$$

which is Newton's interior solution exactly, with the same G. Numerical evaluation of the full integral confirms it:

Table 5: Interior field of a uniform sphere: the model against Newton's interior solution.

r/R	model	Newton	ratio at $\kappa = 10^{-22}$	ratio at $\kappa = 10^{-12}$
0.10	1.170990×10^{-35}	1.170990×10^{-35}	1.000000	0.9655
0.25	2.927474×10^{-35}	2.927475×10^{-35}	1.000000	0.9659
0.50	5.854949×10^{-35}	5.854949×10^{-35}	1.000000	0.9673

r/R	model	Newton	ratio at $\kappa = 10^{-22}$	ratio at $\kappa = 10^{-12}$
0.75	8.782423×10^{-35}	8.782423×10^{-35}	1.000000	0.9698
0.99	1.159280×10^{-34}	1.159280×10^{-34}	1.000000	0.9739

At the centre the columns are equal in every direction, the attenuation is isotropic, and the net force vanishes — as it must.

The opaque regime has no interior field whatever. This is worth stating as a separate refutation, because it is the most intuitive of the three. If a body deflects every particle crossing it, then no particle from outside reaches its interior along a straight line: the inside is completely dark, and every interior point feels no force at all. All of the body's gravitational interaction would take place in an infinitesimally thin surface skin — which is precisely why $S \to \pi R^2$ in that limit — and the entire interior of the Earth would be weightless. The numbers show the interior field switching off as the optical depth rises: at $\tau_{\text{centre}} = 7.0$ the field at mid-radius has fallen to 4.8% of Newton's, at $\tau = 70$ to 2×10^{-10} of it, and at $\tau = 700$ to 4×10^{-19}.

So transparency is not a convenience; it is what makes gravity a volume effect. Because $\kappa\sigma \ll 1$, the G sea floods every body completely, every interior mass element blocks its own share, and three things follow together that would otherwise be separate assumptions: the force couples to mass rather than to surface area (§7.2), a body's interior gravitates at all, and the interior field takes the Newtonian form. They are one fact seen from three sides.

8 Remark: Coulomb's law as the same mechanism one level down

This section is a remark, not a result. It takes postulate P5, the framework's claim that gravitic motion occurs at every level, as given, and asks what §3–§7 then say at level 2. Nothing in it is needed for the level-1 results of §3–§7 and §9, and this paper does not test P5.

8.1 The framework's claim: gravitic motion at every level

The four motions do not place gravitic motion at one privileged scale. The claim is stronger and simpler: **the universe has infinitely many levels, and gravitic motion occurs at every one of them.** At each level there is a sea of particles moving in random directions, and there are bodies immersed in it that block some of what passes through. Name the seas $G1, G2, G3, \ldots$ downward — and, since the levels are unbounded in both directions, upward as well.

The bodies of one level are built from the particles of the next. Ordinary matter blocks G1 and is thereby held in gravitational orbit; the constituents of ordinary matter block G2 and are thereby held in what we have historically called electric attraction; those constituents are themselves built from bodies that block G3. There is no bottom level and no top one, only the same mechanism repeating.

On this reading the electric force is not a second kind of force requiring a second kind of substance. It is **gravity one level down** — the identical shadowing mechanism, in the G2 sea, between G2-scale bodies. Nothing called charge appears anywhere in the ontology.

What licenses this is a property of the derivation itself, and it is worth isolating as a theorem.

Theorem 2 (scale invariance). *The derivation of §3–§5 nowhere refers to a scale. It refers only to a collisionless isotropic sea of intensity ρ and to bodies with blocking areas S_1, S_2. Therefore at every level at which such a sea and such bodies exist, the two-body force is*

$$F \;=\; \frac{\rho}{4\pi}\,\frac{S_1 S_2}{d^2},$$

inverse-square, attractive, bilinear in the blocking areas, and equal and opposite on the two bodies.

No step in §5 used the size of a planet, the mass of anything, or the particles' speed; the cone integral (4) and the solid-angle argument of §5.6 are pure geometry. So, granted P5, the inverse-square law is not only a fact about gravity but a fact about every level at once. **Wherever there is gravitic motion, there is an inverse-square law**; that there is gravitic motion at every level is the framework's assertion, not something shown here.

8.2 The G2 sea and the unified law

Instantiating Theorem 2 at the first two levels:

Table 6: Theorem 2 instantiated at level 1 (gravity) and level 2 (electricity).

	Level 1 — gravity	**Level 2 — electricity**
Sea	G1, speed $v_1 = c$, intensity $\rho_{G1} = n_1 m_{G1} c^2$	G2, speed $v_2 > c$, intensity $\rho_{G2} = n_2 m_{G2} v_2^2$
Bodies	planets, stars, ordinary matter	electrons, protons — G2-scale bodies
Blocking area	$S = \kappa m$ (transparent limit)	$s = q/\sqrt{\varepsilon_0 \rho_{G2}}$
Resulting law	$F = \dfrac{\kappa^2 \rho_{G1}}{4\pi}\dfrac{m_1 m_2}{d^2}$	$F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{d^2}$
Coupling constant	$G = \dfrac{\kappa^2 \rho_{G1}}{4\pi}$	$\dfrac{1}{4\pi\varepsilon_0} = \dfrac{\rho_{G2}\, s_e^2}{4\pi e^2}$

Both rows are the same formula. Only the sea and the blocking areas differ. (The level-1 opacity of §7 is written κ throughout; its level-2 counterpart is κ_2.) Coulomb's law therefore requires no derivation of its own — it is Theorem 2 again, read at level 2.

8.3 Charge is a blocking area in the G2 sea

Matching Theorem 2 at level 2 to Coulomb's law term by term gives the identification directly. Requiring

$$\frac{\rho_{G2}}{4\pi}\,\frac{s_1 s_2}{d^2} = \frac{1}{4\pi\varepsilon_0}\,\frac{q_1 q_2}{d^2}$$

for all pairs of bodies forces

$$\boxed{\;q \;=\; \sqrt{\varepsilon_0\, \rho_{G2}}\;\; s\;} \tag{15}$$

Charge is not a primitive property. It is a body's **blocking area in the G2 sea**, in the same way that mass — through $S = \kappa m$ — is its blocking area in the G1 sea. Charge and mass are the same kind of quantity, measured against two different seas. Dimensional check: $[\varepsilon_0 \rho_{G2}] = \mathrm{C^2 J^{-1} m^{-1}} \cdot \mathrm{J\,m^{-3}} = \mathrm{C^2 m^{-4}}$, so $\sqrt{\varepsilon_0 \rho_{G2}}\, s$ is a charge.

Three consequences follow immediately, and they are not small.

Charge is quantised, necessarily. Blocking areas add in the transparent limit (§7), so (15) makes charge *linear* in blocking area and therefore additive. A body built from N identical constituents has N times the G2 blocking area and hence exactly N times the charge. If the constituents of matter are identical particles, charge comes in exact integer multiples of one unit — not approximately but exactly, and for the same reason that a stack of N identical filters blocks N times as much. In standard physics the exact quantisation of charge is an unexplained empirical fact. Here it is a theorem about counting identical blockers.

Charge additivity is a G2 transparency test. By §7 the additivity is exact only while the constituents are optically thin to the G2 sea; the first correction is the same $\Phi(\tau)$ deficit, of fractional size $\frac{3}{4}\kappa_2 \rho^{\mathrm{col}}$. The neutrality of bulk matter is verified to about one part in 10^{21}, so G2-scale bodies must be even more transparent to the G2 sea than ordinary matter is to G1. That is a definite prediction inherited from §7, not a new assumption.

Charge has no independent existence. Nothing in the model needs a substance called charge, a field, or a mediating boson. What exists at every level is a sea and objects that block it.

8.4 Why the two equations look alike

This is the question the framework claims to answer, and on this account the answer is short.

Newton's law and Coulomb's law have the same form because they are **one law read at two levels**. The $1/d^2$ in both is the same $1/d^2$: a blocking area at distance d removes S/d^2 steradians from the sky, and solid angle in three dimensions goes as the inverse square (§5.6). The products $m_1 m_2$ and $q_1 q_2$ are the same bilinear structure: one factor for the body that casts the shadow, one for the body that catches it. And the 4π appearing in G and in $1/4\pi\varepsilon_0$ is the same 4π — the total solid angle of a sphere, where the flux from all directions was normalised in (1).

That last point deserves emphasis. In SI units the 4π in Coulomb's constant is conventionally explained as a bookkeeping artefact of "rationalised" units. In the shadowing account it is nothing of the kind: it is a physical solid angle, present for the same reason in both laws. The two constants have identical anatomy:

$$G = \frac{1}{4\pi}\,\rho_{G1}\,\kappa^2, \qquad \frac{1}{4\pi\varepsilon_0} = \frac{1}{4\pi}\,\rho_{G2}\left(\frac{s_e}{e}\right)^2. \tag{16}$$

In each case: one over the total solid angle, times the sea's intensity, times the square of a coupling converting a body's "amount" into a blocking area. The resemblance of the two force laws is not a coincidence awaiting explanation — it is the visible signature of one mechanism running at two of its infinitely many levels.

8.5 What the G2 sea must be

The identification (15) fixes one combination of the G2 parameters exactly. For two electrons,

$$\rho_{G2}\, s_e^2 = \frac{e^2}{\varepsilon_0} = 2.899 \times 10^{-27}\ \mathrm{J\,m}. \tag{17}$$

One equation in two unknowns, so the model does not yet predict both — but it converts the electron's charge into a statement about its size and its sea:

Table 7: G2 sea intensity implied by (17) for three candidate electron blocking radii.

assumed electron G2 blocking radius	$s_e = \pi r^2$	implied ρ_{G2}
classical electron radius, 2.82×10^{-15} m	2.49×10^{-29} m^2	4.66×10^{30} J m^{-3}
reduced Compton wavelength, 3.86×10^{-13} m	4.68×10^{-25} m^2	1.32×10^{22} J m^{-3}
Bohr radius, 5.29×10^{-11} m	8.80×10^{-21} m^2	3.75×10^{13} J m^{-3}

Given $\rho_{G2} = n_2 m_{G2} v_2^2$, fixing ρ_{G2} fixes the combination $n_2 m_{G2} v_2^2$, as ρ_{G1} fixes $n_1 m_{G1} c^2$ at level 1. Because $v_2 > c$, a given ρ_{G2} needs less G2 mass per unit volume than the same intensity would at the speed of light.

The strength ratio between the two levels also becomes structural rather than brute. For two electrons the ratio of electric to gravitational force is $k_e e^2 / G m_e^2 = 4.17 \times 10^{42}$, and the model says this equals

$$\frac{\rho_{G2}}{\rho_{G1}} \left(\frac{s_e}{S_e} \right)^2 = 4.17 \times 10^{42}, \tag{18}$$

that is: the electron blocks the G2 sea vastly more effectively than it blocks the G1 sea, and/or the G2 sea is vastly more intense. Both are physically meaningful claims about the constituents of the world. The "hierarchy problem" — why gravity is so absurdly weak next to electricity — becomes a question about two blocking ratios between adjacent levels, which is at least the *kind* of question that can have an answer.

8.6 What is not yet derived: repulsion and neutrality

Shadowing produces attraction and only attraction. Removing flux from a direction can never push a body away from that direction. So gravitic motion at level 2 gives the *form* $q_1 q_2 / d^2$, the quantisation, and the 4π — but not the two signs of electricity. This is the open problem at the heart of §8 and should be stated plainly rather than tidied away.

Two things must be explained that blocking alone cannot:

- **Repulsion.** Like charges push apart. Shadowing has one sign.
- **Neutrality.** A proton and an electron together exert no net Coulomb force on a distant third body. Shadows do not cancel — two shadows always add — so cancellation requires something carrying a sign.

Within the four motions the answer is not a patch but the *second* motion. Magnetic motion is circular motion in which **same rotations repel and opposite rotations attract**: a sign carried by the rotational state of a body rather than by the geometry of the sea. The division of labour is then clean, and is the framework's own structure:

gravitic motion supplies the magnitude and the $1/d^2$ form; magnetic motion supplies the sign.

Concretely this requires a rule by which the rotational states of two bodies modulate their momentum exchange in a collisionless sea — attractive when the states oppose, repulsive when they match, with the modulation equal in magnitude to the shadowing term so that Theorem 2 survives with a sign attached. A spinning body in a particle sea does exchange tangential as well as normal momentum, so a spin-dependent term is not implausible on its face. But no such rule has been written down and checked here, and the ordinary intuition for spinning bodies (Magnus and Bernoulli effects) is unavailable, because those require a *self-interacting* medium and P2 forbids one at every level. This is the sharpest well-posed problem the framework currently faces, and a tractable one: it asks for a single interaction rule, testable by the same simulation method as §5 and §6.

8.7 G3 and beyond

Because Theorem 2 is scale-free, the tower does not stop at G2. Each level n has its sea Gn, moving at speed v_n (faster than v_{n-1}) with intensity $\rho_{Gn} = n_n m_{Gn} v_n^2$, its own bodies with their own blocking areas, and therefore its own inverse-square law with coupling constant

$$k_n = \frac{1}{4\pi}\,\rho_{Gn}\,(\text{blocking area per unit content})^2\,. \tag{19}$$

Gravity and electricity are levels 1 and 2 of an unbounded sequence. Two remarks bound what can responsibly be said about the rest of it.

First, the ordering of the levels is not fixed by anything in this paper. Which sea is finer, and which class of bodies belongs to which level, must be settled by the framework independently; §8.5 constrains only the product $\rho_{G2} s_e^2$, not the two separately, and the same will be true at every level.

Second, a level-3 force would have to account for the nuclear forces being *short-ranged* rather than inverse-square. Within this framework that has a natural reading: postulate P2 — a collisionless sea — is exactly the condition for unlimited range, since it is what keeps shadows from filling in (§2, §9.4). A G3 sea that is *not* collisionless over nuclear distances would have its shadows refilled beyond one mean free path, screening the force at precisely that scale. That is a suggestive fit rather than a derivation, and it is offered here only to mark the direction: in this framework, range is set by the collisionlessness of the sea, and inverse-square behaviour is what a level looks like when its sea is collisionless.

9 Open problems, quantified

A proposal is taken seriously by having its consequences computed, including the uncomfortable ones. Because $v = c$ is a definite commitment rather than a free parameter, several of these are now sharp numbers rather than order-of-magnitude worries.

9.1 What the G1 sea must be

Calibrating (5) on the Earth–Moon system in the opaque regime,

$$\rho_{G1} = \frac{4GM_\oplus M_{\rm moon}}{\pi R_\oplus^2 R_{\rm moon}^2} = 3.04 \times 10^{11}\ \mathrm{J\,m^{-3}}, \qquad n_1\, m_{G1} = \frac{\rho_{G1}}{c^2} = 3.4 \times 10^{-6}\ \mathrm{kg\,m^{-3}}.$$

The G1 sea would carry about 3.4 milligrams of mass per cubic metre — roughly 4×10^{20} times the mean cosmic matter density, and 7×10^{16} times the energy density of sunlight at Earth's orbit. Large, but not absurd on its face, and worth noting that this is *far* less extreme than the transparent-regime alternative requires ($\rho_{G1} \gtrsim 3 \times 10^{37}$ J m^{-3}, or $n_1 m_{G1} \gtrsim 4 \times 10^{20}$ kg m^{-3}, §9.3). The opaque reading is the economical one as far as flux intensity goes.

9.2 Drag, and the interaction that removes it

A body moving at speed u through the sea meets more particles head-on than astern. By P4, each particle the body deflects pushes it with $m_G v$ along the particle's own line of travel; the body's own motion changes neither the size nor the direction of that push (the direction rule). What the body's motion does change is how many particles it meets per second from each direction. For an interaction whose strength does not depend on speed, working to first order in u/v, the retarding force on a body of blocking area S is

$$F_{\rm drag} = \frac{1}{3}\, S\, \rho_{G1}\, \frac{u}{v}, \tag{20}$$

and dividing by the two-body force (8) gives a ratio in which the moving body's own properties cancel:

$$\frac{F_{\rm drag}}{F_{\rm grav}} = \frac{4\pi}{3}\, \frac{u}{v}\, \frac{r^2}{S_\odot}. \tag{21}$$

For the Earth at $v = c$, with $S_\odot = \kappa M_\odot$ and the bound of §9.3, this is 9.4×10^{11}. Because (20) is proportional both to the body's mass and to its speed, the implied stopping time $v/\frac{1}{3}\kappa\rho_{G1}$ is the same for every object at every speed — 5.4×10^{-6} s. Taken at face value the model would forbid inertial motion altogether, and be refuted by a thrown ball rather than by astronomy. This is the objection that has attended push gravity since Laplace [9], who was driven to postulate speeds $\gtrsim 10^8 c$, and Poincaré [10], $\gtrsim 2 \times 10^{18} c$.

The escape is not a larger v; it is how often a particle is deflected. Let the cross-section depend on the relative speed as $\sigma \propto w^{-p}$. The number of particles a body deflects per second from a given direction is proportional to $w\,\sigma(w) \propto w^{1-p}$, so the first-order drag coefficient becomes

$$\frac{1-p}{3}, \tag{22}$$

which vanishes at $p = 1$. The cancellation is not a first-order accident. At $p = 1$ the product $w\,\sigma(w)$ is the same for every direction, so the body receives the same number of pushes per second from every direction, each of the same size and each along its particle's own line. They cancel exactly, at every speed $u < v$.

Gravity is untouched by this. The shadow is a *spatial* anisotropy in the flux, not a kinematic one: a body at rest sees $w = v$ from every direction, so the shadowing derivation of §5 and §7 proceeds unchanged. Only the response to the body's own motion is cancelled.

The time-inside rule. $p = 1$ has a plain collision reason. Whether a G particle crossing a nucleon is deflected depends on how long it spends inside it. For an interaction this weak the chance of deflection is proportional to that time, and the time is inversely proportional to the closing speed, so $\sigma \propto 1/w$. A moving body meets more particles ahead, but each crosses faster and is proportionally less likely to be deflected, and the two cancel. The same dependence is familiar in nuclear physics as the "$1/v$ law" for the capture of slow neutrons, where the chance of capture is set by the time the neutron spends near the nucleus.

How exact it must be. With the drag ratio of 9.4×10^{11} found above for the Earth, and planetary ephemerides limiting any anomalous tangential acceleration to about 1.7×10^{-12} of the gravitational one, the residual $(1 - p) \times 9.4 \times 10^{11}$ must stay below 1.7×10^{-12}:

$$|\,1 - p\,| < 1.8 \times 10^{-24}. \tag{23}$$

This need not be a tuning. If the chance of deflection is strictly proportional to the time spent inside the target, $p = 1$ holds exactly, and departures arise only where that proportionality fails.

Where it fails: self-shielding. The proportionality holds for a single nucleon, but a body is many nucleons deep, and a particle deflected in its outer layers no longer reaches its inner ones along its original line. The blocking area of a whole body is therefore $\pi R^2 \Phi(\tau)$ of (7), not linear in the optical depth, and the second-order term of Φ does not cancel. Direct evaluation gives a residual drag of $\frac{3}{8}\tau$ times the constant-interaction drag (20), with $\tau = 2\kappa\rho R$. For the Earth $\tau = 3.5 \times 10^{-13}$, and the residual is about 0.12 of the Sun's pull on it: far above the ephemeris limit, and enough to stop the Earth's orbital motion within a few years.

This residual does not depend on κ. The optical depth rises with the opacity while the drag ratio (21) falls with it, exactly in proportion:

$$\tfrac{3}{8}\,(2\kappa\rho R) \cdot \frac{4\pi}{3}\,\frac{u}{v}\,\frac{r^2}{\kappa M_\odot} \;=\; \frac{\pi\rho R\,u\,r^2}{v\,M_\odot} \;\approx\; 0.12.$$

Every other number in this section moves with κ; this one does not, so no future measurement of κ can relieve it.

What the cancellation rests on: P4. The cancellation above uses P4 as stated: a deflected particle pushes the body with $m_G v$ along its original line, which is what happens if it leaves the body in a random direction at speed v *in the frame of the sea*. The ordinary reading of an elastic deflection is different: the particle leaves in a random direction at the speed it arrived *in the frame of the body*. The body then keeps the particle's full relative momentum, $m_G(v\hat{\mathbf{n}} - \mathbf{u})$, and the $-\mathbf{u}$ part, summed over every deflection, gives a drag that depends only on how many particles the body deflects per second in total. That drag survives for every p. At $p = 1$ it is three times the constant-interaction drag (20), first order in τ where self-shielding is second order, about 2.8×10^{12} times gravity for the Earth. For a body at rest the two readings are the same statement; for a moving body the difference between them is the entire cancellation. P4 stated for a moving body is the *direction rule*, but a reader who takes "the body keeps the relative momentum" as the standard result will not grant it without that argument.

So the position is this. The drag objection, which has stood against push gravity for two centuries and which no adjustment of ρ_{G1}, κ or v answers, is answered exactly for each thin layer of matter by P4 together with the time-inside rule. Two questions remain, in order of importance. First, P4 for a moving body: more of the framework rests on it than on anything else in this section.

Second, self-shielding, where the framework as it stands conflicts with observation by a margin no measurement of κ can change.

9.3 The opacity of matter, and what it costs

§7.3 settled which regime describes ordinary matter: the transparent one, since the opaque law cannot even fit the solar system to itself. That fixes the operative law as Newton's, with $G = \kappa^2\rho_{G1}/4\pi$, and turns κ into a measurable quantity — because κ is precisely the gravitational-shielding coefficient that has been bounded experimentally since Majorana.

Identifying the two and importing the experimental bound is likewise not new — Nali [7] does exactly this. The tightest limit comes from lunar laser ranging. Eckhardt [11] obtained $\kappa < 2\times10^{-21}$ cm^{2}g^{-1}. The bound comes from a lunar-ranging test of the equivalence principle. Williams, Turyshev and Boggs [12] repeat his analysis on a modern lunar-ranging solution and find $h = (3\pm5)\times10^{-23}$ cm^{2}g^{-1}, consistent with zero. Taking the uncertainty as the bound,

$$\kappa \lesssim 5 \times 10^{-23}\ \mathrm{cm^2 g^{-1}} = 5 \times 10^{-24}\ \mathrm{m^2 kg^{-1}}, \tag{24}$$

about forty times tighter than Eckhardt's. For comparison: Majorana's own experiments (1919–20) gave $\sim 7 \times 10^{-12}$; solar-eclipse observations (1961) $\lesssim 6 \times 10^{-16}$; lunar-motion fluctuations (1974) $\lesssim 6.3 \times 10^{-15}$; the Earth-as-screen method (2002) $\lesssim 2 \times 10^{-17}$ [13]. The crossover to the opaque regime, $\tau = 1$, sits at $\kappa \approx 1.4 \times 10^{-11}$ for the Earth, so matter is about twelve orders of magnitude inside the transparent regime — comfortably consistent with §7.3.

This is the bound that costs the model. Because $\rho_{G1} = 4\pi G/\kappa^2$, a more transparent matter demands a more intense sea, and (24) forces

$$\rho_{G1} \gtrsim 3 \times 10^{37}\ \mathrm{J\,m^{-3}}, \qquad n_1 m_{G1} = \frac{\rho_{G1}}{c^2} \gtrsim 4 \times 10^{20}\ \mathrm{kg\,m^{-3}}, \tag{25}$$

some twenty-six orders of magnitude above the economical value the opaque reading would have allowed (§9.1). In mass terms the G1 sea must carry at least 4×10^{20} kg in every cubic metre of empty space, over a thousand times the density of nuclear matter. It is this figure, not the geometry, that drives the drag and heating problems of §9.2 and §9.4. Because the dependence on κ is quadratic, every future tightening of κ raises it as the square: the factor of forty from Eckhardt's bound to the present one raised it sixteen-hundredfold.

The equivalence principle is not an independent test. The residual opacity produces a size- and density-dependent departure from the equivalence principle, $m_g/m_i = 1 - f\kappa\rho R$ with $f = \frac{3}{4}$ for a uniform sphere — the result of Majorana and of Russell [14], re-derived in (10). For a centrally condensed body Russell's $f = (2n+3)^2/(12n+12)$ for $\rho(r) = \rho_0(1 - r^2/R^2)^n$ applies; Williams, Turyshev and Boggs use $n \approx 0.8$ for the Earth, $f \approx 0.98$. With their values of $f\rho R$ and the bound above, the departures are 2.2×10^{-14} for the Moon and 1.7×10^{-13} for the Earth. These sit at the sensitivity of lunar ranging, but not as a prediction: the bound on κ was obtained *by* a lunar-ranging test of the equivalence principle, dividing the measured difference in the Earth's and Moon's gravitational-to-inertial mass ratios by the difference in $f\rho R$. Presenting the deficit as a test the model faces would count that measurement twice. What the model does predict is the *form* of any future violation: one that scales with $f\rho R$, a body's column density, rather than with its composition. A detection of that form would be a measurement of κ, and hence, through (12), the first direct measurement of the G1 sea.

9.4 Energy deposition

If every G particle that met matter were stopped and its motion turned entirely into heat, the sea of §9.1 acting on the Earth's blocking area $S = \kappa M_\oplus = 30$ m^2 (the transparent regime of §7) would deliver $\frac{1}{2}\rho_{G1} c\, \kappa M_\oplus = 1.5 \times 10^{47}$ W to the Earth, or 2.5×10^{22} W kg^{-1}, some 3×10^{33} times the observed heat flow of 7.9×10^{-12} W kg^{-1}. This is Maxwell's old objection with a modern number attached. It assumes that a particle which pushes a body also sets the body's parts moving in proportion, and the force needs only the push. Elastic deflection off a heavy bound scatterer delivers a push $\sim m_{G1}c$ but sets the scatterer's parts moving only by a fraction m_{G1}/M of it. The deflection is elastic in this sense: the particle leaves at the speed it arrived. The heating problem is solved by elasticity provided the G1 mass is small enough relative to a nucleon-scale scattering centre — a definite requirement the framework can meet or fail to meet, and one worth checking against whatever the four motions independently say about m_{G1}.

Note the trade-off: elastic deflection redirects particles rather than removing them, so the redirected flux partially refills the body's own shadow. Any elastic version must show that the refilled fraction is small or emitted preferentially forward. Postulate P2 already disposes of the *other* half of the shadow-filling worry — with no G1–G1 scattering, nothing fills the shadow in transit, however far the second body lies.

The answer to the heating objection. The oldest and most persistent objection to particle gravity is that a sea intense enough to push the planets together would heat them, and gravity produces no heat. Maxwell gave the objection its classic form in the article "Atom" (*Encyclopaedia Britannica*, 1875) [15], and he framed it, as the estimate above does, in terms of energy: the sea's energy must go somewhere, and the only place for it is heat. The four motions do not accept that framing. Energy is a calculation, a quantity of bookkeeping, not a physical thing; what exists are particles moving and colliding, or passing without colliding. The objection must therefore be answered in terms of what physically happens when a G particle meets matter, and in those terms it fails at every step.

Heat is a frequency. Heat is not motion in general; it is a jiggling of matter, and jiggling requires a rhythmic, repeated push. Thermal radiation is light, and light, in luminic motion, is groups of particles travelling together in the same direction at the same speed, arriving in a regular succession. That regularity is a frequency, and the frequency is what matter registers as heat. This is why heat can be seen at all: an infrared detector senses a band of frequencies, and a body's temperature is read directly from the frequencies it emits.

Gravitic motion supplies no frequency. G particles travel singly, in random directions, with no relation between one arrival and the next. Each hit is a single random push. Pushes from all sides very nearly cancel (§4), and in the transparent regime of §7.3 hits are rare, because any given G particle almost always passes straight through the matter it crosses. A sequence of isolated pushes from random directions at random times has no period, and so it cannot set matter jiggling, however dense the sea. What random motion produces is an overall pressure, and pressure is all that gravity requires. Gravity is felt as a push and never as warmth, which is exactly what is observed: no body has ever been warmed by being in a gravitational field.

The G sea has no container, and therefore no temperature. The rejoinder from kinetic theory is that random molecular impacts *do* warm a body: a hot gas warms the wall of its container. But temperature, as kinetic theory defines it, belongs to particles held in a container — trapped, striking one another and the walls over and over until they settle into balance with them. Gas warming a wall is that settled state being shared. None of this applies to the G sea. G particles have no

container. They fly through unbounded space at the speed of the sea, pass through atoms with ease, almost never collide with matter and never with one another (P2), and so never settle into balance with anything. The sea has no temperature at all, and the picture of a hot sea warming a cold body has nothing to attach to. The same rarity of collision is why gravity is so much weaker than the forces between atoms.

A G-particle hit is too small to make an atom vibrate. A gas warms a wall because atoms strike atoms: projectile and target are of comparable size, bound by the same forces, and each impact sets the target's atoms vibrating. A G particle is an entirely different object, most likely far smaller than an electron and many levels below the atom. The push it delivers on deflection is far smaller than the smallest vibration the atoms of a body can take up. Matter cannot respond to such a push piece by piece; it responds as a whole. The nucleon that is struck is bound in a nucleus, and the nucleus in the body, and the push is carried by the entire body moving together. A push taken up by the whole body is a drift, and drift is gravity; a push taken up by individual atoms is vibration, and vibration is heat. G-particle hits fall entirely on the first side of that line. Laboratory physics knows the same behaviour: in the Mössbauer effect, a recoil too small to excite the vibrations of a crystal is taken up by the crystal as a whole, and the struck nucleus is left without any jiggle at all.

G particles are deflected, not stopped. Finally, nothing is left behind. A G particle that meets matter is deflected, always, and continues on its way at the speed it arrived. The only way one could remain in a body is to be captured, held in orbit about a nucleon, and the chance that a particle moving at the speed of the sea is captured in this way is vanishingly small.

Taken together: the sea delivers single, rare, uncorrelated pushes; it has no temperature to share; each push is too small to make any atom vibrate and is carried by the body as a whole; and the particle leaves again. There is no step at which heat could arise. The heating objection is removed at its root rather than merely suppressed, and any bound on the G1 mass drawn from heating is correspondingly relaxed.

9.5 A preferred frame

The sea is isotropic in exactly one frame, and (20) makes that frame physically detectable. The model is therefore not Lorentz invariant. With $v = c$ this is less severe than in the classical superluminal versions, but it remains a real commitment whose consequences — a detectable absolute rest frame, and frame-dependent simultaneity of gravitational effects — need to be worked out rather than left implicit.

10 Relation to previous work

A framework of this age accumulates literature, and much of what is derived above has been derived before. This section states what is old, what is old but scattered, and what we believe is new — the last with the caveat that our search was not exhaustive.

10.1 The mechanism and the classical objections

The shadowing mechanism is Fatio de Duillier's [16] and Le Sage's [17]. The inverse-square law from solid-angle geometry, the requirement that matter be nearly transparent for the force to couple to mass, and the drag and heating objections are all classical: Laplace [9] on aberration, Maxwell

[15] on thermodynamics, Poincaré [5, 10] on drag and heating with explicit numbers, Kelvin and Darwin on the energy balance. Sections 5 and 9 of this paper are re-derivations. Their only novelty is that they are worked at the definite commitment $v = c$, which sharpens Poincaré's estimate into the concrete drag-to-gravity ratio of 9.4×10^{11} in (21).

The twentieth-century revival includes Radzievskii and Kagalnikova [6], Shneiderov (1961), Buonomano and Engel (1976), Adamut (1982) and Jaakkola (1996) [18]; the collected volume *Pushing Gravity* (Edwards, ed., 2002) [19] contains Martins on the Le Sage–Majorana connection, Slabinski on force, heat and drag, and a reprint of Radzievskii and Kagalnikova.

10.2 Results in §7 that are already published

Three of this paper's quantitative results have prior art, and we state the correspondences explicitly.

The blocking function $\Phi(\tau)$ is Majorana's apparent-mass function. Majorana [4, 20] introduced

$$\Psi(u) = \tfrac{3}{4}\left[\tfrac{1}{u} - \tfrac{1}{2u^3} + e^{-2u}\Big(\tfrac{1}{u^2} + \tfrac{1}{2u^3}\Big)\right], \qquad u = \kappa\rho R,$$

for the ratio of a sphere's apparent to true mass under exponential attenuation. With $\tau = 2u$ our (7) satisfies $\Phi(\tau) = \frac{4}{3}u\,\Psi(u)$ identically — verified numerically to machine precision across five decades of u. Equation (7) is therefore a change of variables on a result more than a century old, and the same function appears in Poincaré's lectures and in Radzievskii and Kagalnikova [6].

The deficit $1 - \frac{3}{4}\kappa\rho R$ is published, including its reading as an equivalence-principle violation scaling with column density. Nali [7] gives it as $M_a = M[1 - \frac{3}{4}kR]$ together with the corresponding Eötvös parameter.

The identification $G = \kappa^2\rho_{G1}/4\pi$ is published, as is the step of equating κ with Majorana's shielding coefficient, importing Eckhardt's bound [11], and inverting to a floor on the flux intensity. Nali [7] carries this through in the same order and reaches a comparable floor. Fedosin [8] gives the equivalent constant in cross-section-per-nucleon form, $G \propto p_g\Phi_0\sigma^2/m_n^2$, with the same σ^2 structure and for the same reason.

10.3 §8 and the hierarchy

The idea that the electric force is the same mechanism at a smaller scale, inside an unbounded hierarchy of levels, is Fedosin's. His *Infinite Hierarchical Nesting of Matter* [21] names a ladder of levels — graons, praons, nucleons, stars, metagalaxies — and asserts the same flux mechanism at each; his 2016 paper [22] derives the Coulomb force from a charged component of the vacuum field. Postulate P5 and §8.1 restate that programme in the four-motions vocabulary.

Two differences are worth recording because they cut against us as much as for us.

Fedosin does not identify charge with a blocking area. In his treatment charge is the ordinary count of elementary charges and the cross-section is a separate constant, entering squared as at level 1. Our (15), $q = \sqrt{\varepsilon_0\rho_{G2}}\,s$, makes charge *be* the blocking area, which is what yields quantisation as a theorem (§8.3). We believe that identification is ours; it is also weaker in one decisive respect, below.

Fedosin obtains both signs; we do not. His vacuum field has positive and negative components, and a charged body attracts one component and repels the other, so that two like charges each focus the opposite-sign flux into the gap between them and are pushed apart. That is a working mechanism

for repulsion, which pure shadowing does not have and which §8.6 concedes we lack. The cost is that the flux must respond to the bodies' fields — arguably assuming part of what is being derived — but it is a cost paid rather than deferred.

10.4 What we believe is new here

Stated conservatively:

1. **The solar-system falsification of the opaque regime.** The effective constant $G_{\text{eff}} = 9\rho_{G1}/64\pi(\rho_1 R_1)(\rho_2 R_2)$ of (11) follows in one line from published equations, but we have not found it written down, and we have found no published comparison across Earth–Moon, Earth–Sun and Sun–Jupiter showing that the saturated law returns three values of G spanning a factor of 446. The physics is not new; the arithmetic and the falsification appear to be.
2. **Direct simulation of the bombardment.** This is the claim we hold with most confidence, having searched for it specifically. Every Le Sage treatment we could verify — Fatio, Le Sage, Majorana, Radzievskii and Kagalnikova, Slabinski, Fedosin, Ivanov [23], Nali — obtains the force *analytically*, by integrating an isotropic flux over solid angle. Danilatos supplies working code [24], but it evaluates the theory's integral equations deterministically by two routes ("bulk" and "surface"); it does not represent individual particles. We have found no instance, anywhere in the push-gravity literature, of the force being obtained by generating many corpuscles with randomly sampled directions and impact points, tracing each one, deciding deflection body by body, and summing the transferred momentum.

That is what §6 and §7.4 do. It is worth being precise about what this does and does not establish, because it is easy to overclaim.

Both routes rest on the same postulates P1–P4, and Monte-Carlo sampling of an integral is still evaluation of that integral. The simulation therefore tests nothing about whether the postulates are true. What separates the two is narrower and worth stating exactly: **the analytic route makes one assumption the simulation does not.** Equation (1) replaces the hail of discrete particles with a smooth isotropic flux of intensity ρ_G — a continuum idealisation, introduced before any force is computed. The simulation never writes (1) down. It samples individual directions and impact points, and isotropy *emerges* from the sampling rather than being imposed on it.

So the simulation independently establishes three things: that the analytic reduction is algebraically correct (the solid-angle integral, the $\cos\theta$ cancellation of (4), the change of variables of §5.4); that the geometric bookkeeping is right (which body lies upstream on each ray, chord lengths, ordering); and — the one of real physical interest — that **the continuum idealisation is faithful**, that treating discrete random bombardment as a smooth flux costs nothing. That last point is not decorative in a framework whose entire ontology is discrete particles: a reader is entitled to object that a derivation which replaces the corpuscles with a continuous field has stopped being a particle theory. Sections 6 and 7.4 answer that objection directly.

The method itself is not without precedent, and the precedents are worth citing because they make it respectable rather than exotic. Shadowing-induced attraction *has* been simulated at particle level in adjacent systems: the depletion force between two large spheres in a bath of small ones [3], granular depletion forces [25], and shadowing forces in dusty plasmas [26]. Worldline Monte Carlo for the Casimir effect [27] uses the same algorithmic template — sample random paths, test which bodies they intersect, sum. And a Le Sage-like attraction has recently been produced experimentally

in complex plasma (Zobnin et al. [28]), though the accompanying molecular dynamics tracks the dust grains rather than the plasma corpuscles that push them. An analytic example exists too: light shadowed between nanoparticles produces an inverse-square attraction [29].

So the honest statement is narrow and, we believe, correct: the shadowing force has been derived analytically many times and evaluated by numerical quadrature once, and shadowing attractions have been simulated at particle level in other physical systems — but the Le Sage force itself appears never to have been obtained by direct stochastic ray-tracing of the flux. What is new here is the instrument, not the physics it confirms.

That instrument is worth more than the confirmation. The analytic route needs spherical bodies and a far-field or thin-body approximation to stay tractable; the simulation needs neither. It extends without modification to arbitrary shapes, to three or more bodies with overlapping shadows, and — most importantly — to the two unsolved problems of §8.6 and §9.2, where the question is whether a proposed interaction rule *produces* a given behaviour rather than what a known rule integrates to. A rule by which spin modulates momentum exchange either yields repulsion under simulation or it does not.

3. **Transparency as a single fact with three faces** (§7.4): that mass-coupling, the existence of an interior field, and the Newtonian form of that field are one consequence and not three assumptions — together with the observation that the opaque regime predicts *no interior field at all*, which we have not seen used as a refutation.

We claim nothing beyond that list, and note that Danilatos [30] independently argues that the mass attenuation coefficient rather than G is the true universal constant and that apparent mass falls below real mass through self-shadowing — a position close to that of §7.3–7.4, which we have been unable to compare in detail because his equations were not retrievable.

10.5 A caution on this section

The Le Sage literature is scattered across venues that are poorly indexed — Apeiron, Physics Essays, the *Pushing Gravity* volume, Zenodo preprints — and several primary sources we would want to check were not obtainable: Russell [14] on Majorana's theory and solar-system dynamics, the Majorana and Radzievskii originals, Martins' chapter on Le Sage and Majorana, and Danilatos's equations. Russell in particular may well contain something close to §10.4's first item. Any priority claim above should be read as provisional.

11 Conclusion

The question this paper set out to answer has a clean and affirmative answer.

Random G-particle bombardment of two bodies produces an inverse-square force, and it does so necessarily. For two spheres of radii R_1 and R_2 separated by d in a G sea of intensity ρ_G, the force is

$$F = \frac{\pi \rho_G}{4} \frac{R_1^2 R_2^2}{d^2}, \qquad \rho_G = n\, m_G\, v^2,$$

equal and opposite on the two bodies, along the line of centres, attractive, and exactly $1/d^2$. The inverse square is not fitted and not asymptotic: it follows from the single geometric fact that a body of blocking area A at distance d removes A/d^2 steradians from the other body's sky, and the integral $\int_{\text{cone}} \cos\theta \, d\Omega = \pi R_1^2/d^2$ holds identically for every separation down to contact. Direct simulation of millions of random G impacts reproduces the law, its magnitude, and its symmetry between the two bodies. Because no step of the derivation names a level — no planet's size, no body's mass, not even the particles' speed — the same result would hold at any level where a collisionless sea and blocking bodies exist: **wherever there is gravitic motion, there is an inverse-square law.** That such levels exist is the framework's claim, which this paper does not test.

Three further results follow from the same derivation. Newton's third law is obtained rather than assumed, from the symmetry of $S_1 S_2/d^2$. Partial transparency changes what the force couples to — radius squared when opaque, mass when transparent, where the transparent limit reproduces Newton's law exactly with $G = \kappa^2 \rho_{G1}/4\pi$ — but leaves the inverse square completely untouched, so the distance law is a robust prediction of gravitic motion independent of how matter and G1 actually interact. Finally, as a marked remark (§8): the framework places gravitic motion at every one of the universe's levels, and the derivation is consistent with that, since it nowhere refers to a scale. If the framework is right, Coulomb's law needs no separate derivation. Electricity is then level 2 — the G2 sea acting on G2-scale bodies — so that charge is a blocking area, $q = \sqrt{\varepsilon_0 \rho_{G2}}\, s$, with no charge substance required anywhere in the model. Charge quantisation follows as a theorem about counting identical blockers; the 4π in $1/4\pi\varepsilon_0$ is revealed as the same physical solid angle as the 4π in G rather than a bookkeeping convention; and the old puzzle of why the two great force laws share their form is answered in the strongest possible way — **they are one equation instantiated at two scales**. That is what the four-motions framework claims for gravitic motion; this paper shows the force law it would deliver, not that the levels exist.

Following the *constant* rather than the force settles which regime is real. In the opaque regime there is no G: the coupling is a pressure, and forcing the law into Newtonian form gives $G_{\text{eff}} = 9\rho_{G1}/64\pi(\rho_1 R_1)(\rho_2 R_2)$, which returns three different values of G spanning a factor of 446 across Earth–Moon, Earth–Sun and Sun–Jupiter. Three centuries of celestial mechanics exclude it without any laboratory. Ordinary matter is therefore in the transparent regime, where G is genuinely constant and genuinely composite: $G = \kappa^2 \rho_{G1}/4\pi$ — a solid angle, a property of matter, and a property of the vacuum. The universality of G becomes a derived consequence of matter's transparency rather than a postulate, every precision test of that universality becomes a measurement of κ, and $\dot{G}/G = \dot{\rho}_{G1}/\rho_{G1}$ exactly, so the stability of G is the stability of the sea.

The open problems are named and quantified rather than deferred. Orbital drag at $v = c$ appeared fatal — a universal 5.4×10^{-6} s stopping time, refuted by a thrown ball — until §9.2, where a cross-section scaling as $1/w$, the time-inside rule, is shown to cancel it *exactly* at all speeds for each thin layer of matter while leaving the shadowing derivation untouched. The heating objection is answered in the framework itself (§9.4): heat is a frequency, carried by the ordered succession of particles that makes up light, and random, uncorrelated G-particle impacts carry no frequency and so produce no heat, only an overall pressure. The sea has no container and so no temperature to share, each hit is too small to make an atom vibrate and is carried by the body as a whole, and G particles are deflected rather than stopped. That cancellation rests on P4 holding for a moving body, which must be argued rather than assumed, and it fails for a body thick enough to shield its own interior, leaving a drag of about 0.12 of gravity for the Earth whatever the value of κ. These are now the framework's central questions; the opaque-versus-transparent fork is a genuine, testable question the model itself poses; and the sign of the electric force awaits an explicit rule from magnetic motion. Each is a definite target rather than a vague objection — which is the condition a theory should be

in.

A Notation

Table 8: Notation.

Symbol	Meaning	Units
n	number density of G particles at the level in question	m^{-3}
m_G	mass of a G particle	kg
$\rho_G = nm_Gv^2$	G flux intensity — the generic sea, any level	$\mathrm{J\,m}^{-3}$
v	G-particle speed at the level in question	$\mathrm{m\,s}^{-1}$
v_n	G-particle speed at level n; $v_1 = c$ and $v_1 < v_2 < v_3 < \cdots$	$\mathrm{m\,s}^{-1}$
R_1, R_2	radii of the two bodies	m
d	separation of centres	m
$\theta_{\max}$	angular radius of the blocked cone, $\sin\theta_{\max} = R_1/d$	rad
κ	opacity of a body to its level's sea; at level 1, the Majorana coefficient	$\mathrm{m}^2\mathrm{kg}^{-1}$
$\tau = 2\kappa\rho R$	central optical depth of a sphere	—
$\Phi(\tau)$	blocking efficiency, $1 - \frac{2}{\tau^2}[1 - (1+\tau)e^{-\tau}]$	—
$S = \pi R^2\Phi(\tau)$	effective blocking area, any level	m^2
$\rho_{G1} = n_1 m_{G1} c^2$	G1 flux intensity — level 1, gravitation	$\mathrm{J\,m}^{-3}$
$\rho_{G2} = n_2 m_{G2} v_2^2$	G2 flux intensity — level 2, electricity	$\mathrm{J\,m}^{-3}$
ρ_{Gn}	flux intensity of the Gn sea, level n	$\mathrm{J\,m}^{-3}$
s	blocking area of a body in the G2 sea	m^2
$q = \sqrt{\varepsilon_0\rho_{G2}}\,s$	charge — derived, not primitive	C

B Verification code

- `g1.py` — the exact cone integral (4) to twelve digits from contact to $d/R_1 = 5000$; the Monte-Carlo bombardment of two opaque spheres validating (5) and the symmetry (6); and the ρ_{G1}, drag and heating numbers of §9 at $v = c$.
- `radii.py` — the closed form (7) against direct quadrature; the table of $\Phi(\tau)$; and a Monte-Carlo exact in both bodies (full attenuation through body 1, self-shielding within body 2, correct ray ordering) validating (8) in the opaque, mixed and transparent regimes.
- `breach.py` — the drag integral for a cross-section $\sigma \propto w^{-p}$ under P4: the coefficient $(1-p)/3$, the exact vanishing at $p = 1$, the self-shielding residual $\frac{3}{8}\tau$, and the fine-tuning bound (23).
- `interior.py` — the interior-field integral (13)–(14), its agreement with Newton's interior solution across r/R and across κ, and the switching-off of the interior field as the body becomes opaque.
- `coulomb.py` — the level-2 identification $q = \sqrt{\varepsilon_0\rho_{G2}}\,s$, the constraint $\rho_{G2}s_e^2 = e^2/\varepsilon_0$, the candidate G2 blocking radii of §8.5, and the two-level strength ratio (18).

- `verify.py`, `constraints.py` — the transparent-limit derivation, its shell-theorem exactness, and the shielding/equivalence-principle numbers of §9.3.

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